

EFFECT OF A MAGNETIC FIELD ON THE ELECTROWEAK SYMMETRY

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We discuss the effect of a strong magnetic field in the behavior of the symmetry of an electrically neutral electroweak plasma. We analyze the case of a strong field and low temperatures as compared with the W rest energy. If the magnetic field is large enough, it is self-consistently maintained. Charged vector bosons play the most important role, leading only to a decrease of the symmetry breaking parameter, the symmetry restoration not being possible.

I. INTRODUCTION

The Standard Model of the electroweak interaction at finite temperatures predicts the existence of two phases : the symmetric and the broken one at temperatures, respectively, above and below some critical value T_c [1]. The possibility of symmetry restoration under a large magnetic field for this model was firstly considered by Linde [2], who observed that at zero temperature, an increase of the field increases the symmetry breaking parameter. He notices that only the vector particle contribution may lead to the symmetry restoration at sufficiently large field.

The characteristic field, which can substantially modify the symmetry breaking parameter was estimated to be of the order of $B_{ch} \sim 10^{27}G$. Later Ambjorn and Olesen [3] realized that the usual electroweak vacuum become unstable for some critical field value $B_c = \frac{m_w^2}{e} \sim 10^{24}G$, due to the presence of charged vector bosons W. They also obtained a static magnetic solution of the classical electroweak equations, corresponding to a vacuum condensate of W and Z bosons, for $B > B_c$ [4].

The problem was considered in [5] from the quantum statistical point of view. It was concluded that for $B \rightarrow B_c$, the population of the W-ground state increases, leading to a self-magnetization of the system; actually this prevents B to reach the critical value B_c .

We explore the possibility that m_w^2 would decrease, via ξ , with increasing B . We keep in mind the analogy with superconductivity or either, between the symmetry breaking parameter ξ and a Bose-Einstein condensate. The condensate is destroyed by a sufficiently large magnetic field (the critical Schafroth field [6]). We will conclude in the present case that there is actually a decrease of $\xi = \xi(B)$ for increasing B , but we find that the decrease is a small fraction of $\xi(0)$.

We study our problem in the frame of quantum statistics, taking in mind possible consequences for astrophysics and cosmology. We consider the lepton sector of an electrically neutral electroweak plasma. We evaluate the variation of the symmetry breakdown parameter in the external field and examine the possibility of symmetry restoration. The present paper is a summarized and up-to-date version of the previous one [7].

II. THE ELECTROWEAK PLASMA

Thermodynamical properties of an electroweak plasma in a constant magnetic field can be studied if we know the effective potential associated to the system. We obtain this potential for the leptonic sector of the plasma, in the one loop approximation, starting from the Weinberg -Salam model [8], and it has the form

$$V = V_e + V_w + V_h + V_z + V_A + V_\nu + V_t, \quad (1)$$

where the first two terms are related with charged particles (electrons and W bosons). The terms due to the neutral vector boson Z, the electromagnetic field and Higgs scalar are V_z, V_A and V_h , respectively. The neutrinos contribution is V_ν . Finally, $V_t = \frac{\lambda_2}{4} \left(\frac{\xi^2}{2} - a^2 \right)^2$ is the tree effective potential term, which depends on the symmetry breaking

parameter ξ (λ_2 is the scalar coupling constant and a is the "negative mass" parameter). The particle masses are related to the mentioned parameter by the usual expressions $m_w = \frac{g}{2}\xi$, $m_z = \frac{1}{2}\sqrt{g^2 + g'^2}\xi$, $m_\sigma = \sqrt{\frac{\lambda_2}{2}}\xi$, $m_e = \lambda_1\xi$, where g , g' , λ_1 and λ_2 are, respectively, the electroweak, Yukawa and Higgs scalar coupling constants. The value of the symmetry breaking parameter is obtained from the extremum condition, determining the temperature-dependent mass shell,

$$\partial V / \partial \xi = 0. \quad (2)$$

By evaluating the effective potential on the mass shell, we get the thermodynamical potential $V(\xi_{\min}) = \Omega$. One can write then two other equilibrium equations. One of them is the lepton number conservation $-\partial\Omega/\partial\mu_2 = N_l = N_e + N_\nu$ (where N_i is the net density of particles (particles minus antiparticles), per unit volume) and the other $\partial\Omega/\partial\mu_1 = 0$, is the electric charge conservation, which in our simplified model is reduced to $N_e + N_w = 0$. The magnetization $M = -\partial\Omega/\partial B$ contains the contributions of both electrons and W bosons $M = M_e + M_w$.

III. THE STRONG MAGNETIC FIELD LIMIT: SYMMETRY ANALYSIS

In the absence of field and at zero temperature, the effective potential coincides with the tree term $V|_{T=0, B=0} = V_t$, and therefore the equation (2) has only one stable solution $\xi_o = \sqrt{2}a$. It is known that temperature modify the symmetry breaking parameter. In fact, the Higgs model predicts [2] that an increase of temperature decreases the symmetry breaking parameter and at some critical temperature T_c the symmetry is restored ($T_c \sim 10^{15}K$ [2]). We can expect then that an intense external magnetic field also modifies the symmetry breaking parameter.

We will restrict ourselves to the case of a strong magnetic field and/or low temperatures, when the condition $eB \gg T^2$ is satisfied. It can be demonstrated that, in our case, only the charged boson contribution may substantially modify the symmetry breaking parameter (see [7]). For $eB \gg T^2$, the average W boson population in excited Landau states is negligible small. Moreover, in that limit the Bose-Einstein distribution degenerates in a Dirac δ function and the most of the W density is in the Landau ground state $n = 0$ and distributed in a very narrow interval around $p_3 = 0$ [9]. If we only consider the contribution of the W boson sector [1], Eqs. (1) and (2) takes the form [2] (with $C = \frac{gN_w}{\sqrt{2}\lambda_2 a^3}$)

$$V = \frac{\lambda_2 a^4}{4} [(y-1)^2 + 4C\sqrt{y-z}], \quad (3)$$

$$[(y-1)\sqrt{y-z} + C]\sqrt{y} = 0. \quad (4)$$

It can be shown that for fields less than some value $B_m(N_w)$ (that is, for $z < z_m = 1 - 3\sqrt[3]{2C^2}/2$), there is a non zero symmetry breaking parameter ξ , which corresponds to the minimum of the effective potential. When the field grows, ξ decreases, and for fields equal or greater than B_m ($z \geq z_m$), the effective potential will not have a stable equilibrium point. That is, the broken solution symmetry is no longer true. On the other hand, the solution $\xi = 0$ is unrealistic, leading to purely imaginary physical quantities: i.e. if $B \neq 0$ and , we have $\varepsilon_0(p_3 = 0) = \sqrt{-eB}$.

Actually, for solving our problem, we must take into account that for sufficiently large magnetization ($M \gg H$) it can self-consistently maintain the field B . So, we can put $B = 4\pi M$, where $M_w = eN_w/2\sqrt{m_w^2 - eB}$, and consider this equation together with Eq.(4), and by calling $D = FC$, $F = 8\pi e^2\lambda_2/g^4$ we have,

$$z\sqrt{y-z} - D = 0. \quad (5)$$

We may express the solutions z and y as functions of $x_i = \frac{2}{\sqrt{3}} \cos \frac{\kappa_i \pi - \arctan \sqrt{\frac{4}{27E^2} - 1}}{3}$ (with $E = C + D$):

$$z_{1,2} = F \frac{1 - x_{1,2}^2}{1 + F}, \quad y_{1,2} = \frac{F + x_{1,2}^2}{1 + F}. \quad (6)$$

[1] For the W-sector, the effective potential $V_w = V_w^{st} + V_w^o$, where the first term is the statistical part and the second one is the Euler-Heisenberg vacuum term. We can consider $V_w \approx V_w^{st}$ (see [7]).

[2] We write $\frac{\xi^2}{\xi_o^2} = y$, $\frac{eB}{m_w^2} = \frac{z}{y}$, where $y > 0$, $z > 0$ and $z < y$, because we are considering fields $B < B_c$.

where $i = 1, 2$; $\kappa_i = 5, 1$ respectively, and $0 \leq x_1 \leq 1/\sqrt{3}, 1/\sqrt{3} \leq x_2 \leq 1$. Due to the fact that E is proportional to N_w , we deduce that for each N_w , there are two possible values of the symmetry breaking parameter $y_{1,2}$. It is easy to prove that y_1 corresponds to an unstable equilibrium point of the potential, while V has a minimum for $y = y_2$. We conclude, thus, that as N_w grows, the field also grows and the symmetry breaking parameter decreases to the minimum $\xi_{\min} = \kappa \xi_o$, where $\kappa = \sqrt{(1+3F)/3(1+F)}$ and $F = 4\pi \sin^2 \theta_w \eta^2$, where $\eta = m_\sigma/m_w$ and θ_w is the Weinberg angle. Thus, $\kappa (\leq 1)$ increases with increasing η^2 . We take for the Higgs mass the lower bound of 114.4 GeV, according to recent estimates [10] from Particle Data Group. Then $\eta \geq 1.42$, and one gets $F \geq 5.65$, $1 > \kappa \geq 0.949$, which means a 5 per cent reduction of the symmetry breaking parameter. The corresponding density is $N_w \simeq 4 \cdot 10^{47}$. Fixing η , any further increase of the N_w , would not lead to real solutions of V . This means that excited W Landau states start to be populated. But these states contribute diamagnetically to the total magnetization, and therefore, it is kept $4\pi M = B < B_c$. Higher order corrections do not change the essence of the symmetry behavior in the present problem [7].

IV. CONCLUSIONS

We conclude that for a neutral electroweak plasma in a large constant magnetic field, only the charged vector particle contribution may substantially modify the symmetry breaking parameter. For high values of the field, it is maintained self-consistently and the field never reaches its critical value B_c . The symmetry breaking parameter is decreased some amount under the action of the magnetic field, and in consequence, the masses of electrons, W and Z bosons, and Higgs particles become slightly smaller than in the zero field case at zero temperature.

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